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Title: Lie groups in finance and actuarial mathematics

Abstract: The Hull-White (1987) model, where the logarithm of a stock price is modelled as a Brownian motion whose volatility parameter obeys an additional Brownian motion, is shown to have a natural interpretation as Brownian motion on the so-called Heisenberg group. In physical language, this is the Lie algebra generated by the location and the velocity operators and reflecting the Heisenberg uncertainty principle. Models of higher order, where the volatility is itself an iterated stochastic integral of several one-dimensional Brownian motions have an analogous interpretation in the framework of simply connected Lie groups which are nilpotent of higher step. It has been an open question for several decades if the law of the embedding Lévy process of a probability measure on such a group is uniquely determined (if it exists). Recently we have shown that for gaussian measures on the group this is indeed the case. In statistical terms, this means that in such a generalized Hull-White model, the joint law of all occurring one-dimensional Brownian motions in the volatility and the asset price itself at some fixed time point (which can be observed e.g. by repeated sampling from several i.i.d. models) is a sufficient statistic for the whole model among all models with iterated stochastic integrals of one-dimensional Levy processes. A similar result can be proved for the affine group. This shows that e.g. the joint law of a random payment stream obeying a Levy process in a model with random time-dependent force of interest and the latter at some fixed time point is a sufficient statistic for the joint law of the payment stream and the force of interest (i.e. for the accountholder's investment policy depending on the development of the interest level).